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AUTHOR Harshbarger, Ronald J.; Upshaw, Jane
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ABSTRACT

The University of South Carolina at Hilton Head offers a major in Hotel, Restaurant, and Tourism Administration (HRTA). Many students in the HRTA major take both the HRTA Tourism course and a business calculus course. An interdisciplinary research project was designed in an effort to interest these students in the power of calculus as a mathematical discipline linked to their field of study. The goals were to have students identify real world data collected in an organization or business, to model the data as a continuous function, and to answer questions that would be useful in making business decisions. Students investigated the relationship between hotel room pricing and occupancy during off-peak season. (DDR)

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ED 460 004

THE USE OF MATHEMATICAL MODELING IN AN INTERDISCIPLINARY SETTING

Ronald J. Harshbarger, Ph.D

Jane Upshaw, Ph.D.

University of South Carolina Beaufort

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The hotel and restaurant sector of today's global economy are among the fastest growing in the world. South Carolina is the third leading vacation destination in the United States and tourism is one of South Carolina's growing industries. Tourism contributes over $1 billion dollars to Hilton Head Island economy each year. The University of South Carolina offers a major in Hotel, Restaurant, and Tourism Administration (HRTA), which has been offered at the Hilton Head Campus for the last two years. The program has grown to over 40 majors in that short period of time, with students attending from 24 different countries. This Fall, many students in the HRTA major were taking both the HRTA Tourism course and a business calculus course. In an effort to interest these students in the power of calculus as a mathematical discipline linked to their field of study, an interdisciplinary research project was designed. The goals were to have students identify real-world data collected in an organization or business, to model the data using a continuous function, and to answer questions that would be useful in making business decisions. The project assigned was to investigate the relationship between hotel room pricing and occupancy during the off-peak season (November through February). The problem was chosen because of the inherent questions for hotel managers in a resort location such as Hilton Head Island. Because the majority of the HRTA students work in hotels during internship programs, it was an interesting problem for these students to investigate.

Initially, students in the HRTA course were asked to discuss how resort hotels on Hilton Head Island could increase their occupancy rate, revenue, and profit during the off-season. Students made several suggestions about how to increase occupancy, including offering special conference packages. The most popular suggestion was to offer rooms at lower prices than are currently being used. When asked what a competitive price would be, the students gave only guesses, or prices they had heard offered elsewhere. Because many of the students are International students, most of their guesses were not very good.

After these guesses were recorded, students were instructed to gather data regarding room prices and occupancy from the resort hotels on Hilton Head Island. This data was to form the basis for the solution of the problem. At first, students had difficulty collecting the data because it was considered confidential information by the management of some of the hotels. So some students gathered information from existing publications such as the South Carolina Barometer and the Star Report. (No, not that Starr Report.) We were later able to guarantee that the data would not be related to individual hotels, and we were able to collect information directly from the hotels rather than use published data.

Students in the Tourism class were asked to make estimates, without developing formulas, of what room price would maximize the occupancy rate, daily revenue, and profit. They knew that if occupancy was related to daily room price and if the resort hotels had accurately made a connection between room price and occupancy, they could set the room price so that occupancy would remain nearly constant in the off-season.

Table 1 and the scatterplot in Figure 1 show that the relationship between room price and occupancy is not as linear as might be expected; there is wide variation in the distribution, especially for rooms costing more than $100 per day. Students in the HRTA

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course did not reach any useful conclusions about pricing from looking at the table and scatterplot.

TABLE 1

Daily Room Price	Occupancy Rate (%)	Daily Room Price	Occupancy Rate (%)
110	67	119	65
120	47	79	50
100	60	89	52
120	40	69	75
115	38	39	65
75	55	39	45
65	50	45	50
70	52	62	47
44	39	67	47
38	21	36	27

The scatterplot of the data from the table is shown in Figure 1.

Figure 1

Students in the business calculus class were asked to find a model for the data. They felt that this data might be best modeled by a cubic equation, with occupancy rate as a function of room price. Using graphing calculators gave the following cubic equation.

$$y = -0.000002427x^3 + 0.006765x^2 + 1.24466x + 1.5417$$

The graph of this model is shown in Figure 2.

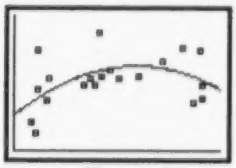

Figure 2

The very small coefficient of x^3 and the shape of the graph over the interval under

consideration led the students to use a quadratic model. The resulting model was

$y = -0.007327x^2 + 1.2865x - 0.6871$.

The graph of this model is shown in Figure 3.

Figure 3

Maximization techniques were used to find the maximum occupancy rate, as follows.

$y' = -0.014654x + 1.2865$

Setting $y' = 0$ and solving for x gave the critical value x = 87.79 for the function.

The students verified graphically and with the second derivative test that a relative maximum occurs at x = 87.79. The graph in Figure 3 confirms that this relative maximum is the absolute maximum in the interval under consideration. The maximum occupancy for this group of hotels is 57%, when the room price is $87.79, if this model is accurate for the group.

The conclusions from the calculus course were given to the HRTA class for their consideration. They eventually raised the question of whether maximizing occupancy would necessarily maximize revenue or profit. So they suggested that the calculus class determine the price, if it existed, that would maximize revenue. To do this, the data was considered to represent a "typical" hotel from the group, with 200 rooms. The occupancy rate was then converted into an occupancy by multiplying the occupancy rate (as a percent) times the 200 rooms. They then multiplied each daily room price by the corresponding occupancy to get points relating room price and revenue. This data is shown in Table 2 and the scatterplot of Figure 4.

TABLE 2

Daily Room Price	Occupancy	Daily Revenue	Daily Room Price	Occupancy	Daily Revenue
110	134	14,740	119	130	15,470
120	94	11,280	79	100	7900
100	120	12,000	89	104	9256
120	80	9600	69	150	10,350
115	76	8740	39	130	5070
75	110	8250	39	90	3510
65	100	6500	45	100	4500
70	104	7280	62	94	5828
44	78	3432	67	94	6298
38	42	1596	36	54	1944

Figure 4

The scatterplot shows that a cubic equation might be used to model the revenue as a function of daily room price. Students found that the best cubic equation that fits the data was

$$y = -0.02445x^3 + 4.8400x^2 - 159.3598x + 3442.0935$$

Figure 5 shows the graph of this model.

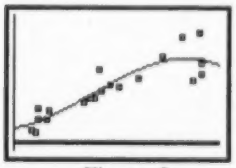

Figure 5

The students then used the derivative to find the value of x that gives maximum revenue.

$$y' = -0.07335x^2 + 9.6800x - 159.3598$$

Setting $y' = 0$ and solving for x gives critical values x = 19.28 and x = 112.69.

The second derivative test and the graph of the function in Figure 5 show that x = 112.69 gives a relative maximum, and the graph shows that revenue is maximized at this value. This maximum revenue is $11,958.

But what about profit, which is really what hotel managers are concerned about? Using a

daily fixed cost of $2450 and a variable cost of $14 per occupied room, the profit function is

P(x) = R(x) - C(x) = Revenue - [2450+14(occupancy)]

To find the function that gives the number of rooms occupied, the students converted the occupancy rate to a decimal by dividing by 100, then multiplied by 200. That is, they multiplied the function that modeled occupancy rate by 2 to get the function that modeled rooms occupied.

$$OCC = -0.014654x^2 + 2.5730x - 1.3742$$

Substituting this into the equation for profit and simplifying gives the profit function

$$P(x) = -0.02445x^3 + 5.0452x^2 - 195.3818x + 1011.3323.$$

The graph of this profit function is given in Figure 6.

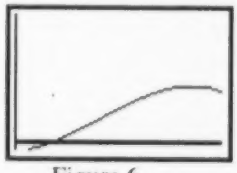

Figure 6

They found the room price that maximizes profit using the derivative.

$$P'(x) = -0.07335x^2 + 10.0904x - 195.3818$$

Setting $P'(x) = 0$ for x gives x = 23.31 and x = 114.25.

Using the second derivative test shows that a relative maximum occurs at x = 114.25, and the graph, shown in Figure 6, shows that this value gives the maximum profit in the interval under consideration. The maximum possible daily profit for this "typical" hotel is $8082.

The conclusions of the calculus class were given to the HRTA class for further discussion. They investigated whether the relatively high daily room price would lower occupancy too much, resulting in loss of revenue in other operations at the hotels, including bars and restaurants. They decided that the occupancy rate did not change sufficiently to affect other operations. They did note that there is a wide disparity of occupancy rate, and profit, when daily room prices are above $100, so they are currently investigating why some hotels have kept occupancy high when others charging the same price have not.

The student response to this collaborative learning strategy has changed the attitudes of many students in the Tourism and the business calculus classes. While they understand the limits of using continuous functions to model data, and that the models may not always fit the data well, they also see that just observing data patterns may not give all the information they need to solve management problems.